<span style="color:blue">**Unedited Draft**</span>

**Arithmetic Revisited**

**Lesson 5:**

**Decimal Fractions or Place Value Extended**

<span style="color:green">**Part 4:  Dividing Decimal Fractions, Part 1**</span>

## <span style="color:darkred">1.  Introduction</span>

So far we have discussed the operations of addition, subtraction and multiplication of decimal fractions and we saw that once we knew where to place the decimal point in the answers, these operations were identical to their counterparts for whole numbers.  In this Lesson we shall see that division works the same way.  Let's first start with the simplest case in which we divide one whole number by another[1].  More specifically, let's start with an example in which the divisor is a factor of the dividend.  For example, based on our adjective/noun theme the fact that $300 \div 4 = 75$ allows us to draw conclusions such as:

$$300 \text{ apples} \div 4 = 75 \text{ apples}$$

If we wanted to use the usual division algorithm ("recipe") to arrive at this result we might first have written

$$
\begin{array}{r}
7 \quad 5 \\
\hline
4 \, | \, 3 \quad {}^3 0 \quad {}^2 0
\end{array}
$$

and then added the noun "apples" to both the dividend and the quotient.  That is:

$$
\begin{array}{r}
7 \quad 6 \quad \text{apples} \\
\hline
4 \, | \, 3 \quad {}^3 0 \quad {}^2 0 \quad \text{apples}
\end{array}
$$

To help get to the main point of this Lesson, let's simply replace the word "apples" above  by the word "hundredths.  When we do this we obtain:

$$
\begin{array}{r}
7 \quad 5 \quad \text{hundredths} \\
\hline
4 \, | \, 3 \quad {}^3 0 \quad {}^2 0 \quad \text{hundredths}
\end{array}
$$

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[1]Notice that every whole number can be represented by a decimal fraction.  For example, the whole number 6 can be written as 6.0 which is a decimal fraction.  This is the decimal equivalent of saying that every whole number can be represented by a common fraction.  For example, $6 = 6 \div 1 = \frac{6}{1}$

Recalling that in place value notation

75 hundredths $= 0.75$ and 300 hundredths $= 3.00$

we may rewrite

$$\begin{array}{r} 7 \quad 5 \quad \text{hundredths} \\ \hline 4 \overline{\smash{\big)}\, 3 \quad 0 \quad 0 \quad \text{hundredths}} \end{array}$$

in the form

$$\begin{array}{r} 0. \quad 7 \quad 5 \\ \hline 4 \overline{\smash{\big)}\, 3. \quad 0 \quad 0} \end{array}$$

In summary, the above algorithm is simply another way of writing

300 hundredths $\div\, 4 = 75$ hundredths

or, equivalently

$3.00 \div 4 = 0.75$

> **More generally, when we use the division algorithm to divide a decimal by a whole number, we simply place a decimal point in the quotient directly above where it appears in the dividend and then divide exactly as we would have done if there had been no decimal point in the problem.**

### Practice Problem #1

Write the quotient $3.36 \div 4$ as a decimal fraction.

**Answer: 0.84**

**Solution:**

**Step 1:**

Rewrite $3.36 \div 4$ as

$$4 \overline{\smash{\big)}\, 3 \quad .3 \quad 6}$$

**Step 2:**

Place a decimal point in the quotient so that it is aligned with the decimal point in the dividend:

$$\begin{array}{r} . \\ \hline 4 \overline{\smash{\big)}\, 3 \quad .3 \quad 6} \end{array}$$

**Step 3:**

Then perform the "usual" division (that is, as if there were no decimal point at all)

```
        .8   4
   4 | 3  .3  6
```

**Note:**

What we actually did when we used the division algorithm was the same as if we had viewed 3.36 as 336 hundredths. In other words, $3.36 \div 4$ means the same thing as 336 hundredths $\div 4$; and since $336 \div 4 = 84$, 336 hundredths $\div 4 = 84$ hundredths.

**Practice Problem #2**

Write the quotient $3.128 \div 8$ as a decimal fraction.

**Answer:  0.391**

**Solution:**

We know that, $128 \div 8 = 3$. Therefore:

3,128 thousandths $\div 8 = 391$ thousandths $= 0.391$

***Notes*:**

• Written in the form of the division algorithm, we line up the decimal points and proceed as if it were a whole number problem,  For example:

```
        0●  3   9   1
   8 | 3●  1   2   8
       2   4
           7   2
       −   7   2
                   8
               −   8
```

• Recall that division is still "unmultplying". In other words, $3.128 \div 8$ means the number which when multiplied by 8 yields 3.128 as the product. Thus we can check our result by observing that $0.391 \times 8 = 3.128$

**Practice Problem #3:**

**Express the quotient 0.000012 ÷ 6 as a decimal.**

**Answer: 0.000002**

**Solution*:***

**One way to solve this problem is to begin with the fact that $12 \div 6 = 2$.  With this in mind we may rewrite the decimal 0.000012 in the form 12 millionths.  Thus:**

$$
\begin{aligned}
\textbf{0.000012} \div \textbf{2} \quad &= \quad \textbf{12 millionths} \div \textbf{6}\\
&= \quad \textbf{(12} \div \textbf{6) millionths}\\
&= \quad 2 \textbf{ millionths}\\
&= \quad \textbf{0.000012}
\end{aligned}
$$

**Notes:**

• **Keep in mind that there are other ways to do this problem.  In particular, if you prefer to work with common fractions, simply rewrite 0.000012 as $\frac{12}{1,000,000}$, whereupon the problem becomes:**

$$\frac{12}{1,000,000} \div 6 \textbf{, or}$$

$$\frac{12}{1,000,000} \times \frac{1}{6} \textbf{, or}$$

$$\frac{2}{1,000,000} \textbf{,}$$

**which in decimal form is 0.000002**

• **Notice that even though $\frac{2}{1,000,000}$ can be reduced to $\frac{1}{500,000}$, there is no need to do this if we want to express the answer as a decimal. In other words, in decimal notation, the "denominators" *must be powers of 10*.**

## Practice Problem #4

Write the quotient 0.0021 ÷ 7 as a decimal fraction.

**Answer:  0.0003**

**Solution**_:_

One way to solve this problem is to begin with the fact that
21 ÷ 7 = 3.  With this in mind we may rewrite the decimal 0.0021 in
the form 21 ten-thousandths.  Thus:

$$0.0021 \div 7 \quad = \quad \text{21 ten-thousandths} \div 7$$
$$= \quad (21 \div 7) \text{ ten-thousandths}$$
$$= \quad \text{3 ten-thousandths}$$
$$= \quad 0.0003$$

**Notes:**

• We could also have used the division algorithm and solve the
problem by writing:

```
        0• 0  0  0  3
    7 [ 0• 0  0  2  1
              −  2  1
                    0
```

• Or if you still are uncomfortable with using the decimal points, the
above algorithm could have been written in the equivalent form:

```
            3   ten thousandths
    7 [ 2  1   ten thousandths
```

## 2.  Dividing By a Non Whole Number Decimal Fraction

One of the nice things about mathematics from a purely logical point of view is that it gives us practice in reducing new problems to equivalent problems which we were able to solve previously.  In the present context, we now know how to divide a decimal fraction by a whole number.  Let's suppose we were then asked the following question.

**Express the quotient 0.000012 ÷ 0.006 as a decimal fraction .**

Here's one approach for solving this problem.  We already know how to divide a decimal by a whole number.  We also know that if we multiply both numbers in a division problem by the same (non-zero) number we obtain an equivalent ratio.

With this in mind we notice that to convert  0.006 into a whole number we must move the decimal point (at least) 3 places to the right.  Shifting the decimal point 3 places to the right means that we have multiplied the decimal by 1,000.  Hence to keep the quotient the same, we must also multiply the dividend (that is 0.000012) by 1,000.

*In summary, then we may move the decimal point 3 places to the right in both 0.000012 and 0.006 to obtain the equivalent problem 0.012 ÷ 6; <u>which we already know how to solve</u>.  Namely :*

*0.012 ÷ 6 = (12 thousandths ÷ 6 = 2 thousandths = 0.002*

**Notes:**

• **Notice that $0.000012 \div 0.006$ does not look like $0.012 \div 6$ but the two quotients express the same ratio.[2]   One way to check this is to do both calculations on a calculator and see that the answers are the same.**

• **As a review of converting decimal fractions to equivalent common fractions, we could also have done this problem by rewriting each decimal fraction as an equivalent common fraction.  More specifically,**

$$0.000012 = \frac{12}{1,000,000} \text{ and } 0.006 = \frac{6}{1,000}$$

**Hence:**

$$0.000012 \div 0.006 \quad = \quad \frac{12}{1,000,000} \div \frac{6}{1,000}$$

$$= \quad \frac{12}{1,000,000} \times \frac{1,000}{6}$$

$$= \quad \frac{2}{1,000}$$

$$= \quad 0.002$$

• **By moving the decimal points the "right" way, we can write the problem as the quotient of two whole numbers.  For example, to make 0.000012 a whole number we would have to move the decimal point 6 places to the right and to make 0.006 a whole number we would have to move the decimal point 3 places to the right.  However, to keep the ratio the same we have to move the decimal point the same number of places in each number.   Thus we would move each decimal point 6 places to the right to get the equivalent problem $12 \div 6,000$ or 0.002. That is:**

```
                    0.  0   0   2
          6,000 | 12.  0   0   0
                  12.  0   0   0
```

• **Again, If we wanted to use common fractions we could write $12 \div 6,000$ as $\frac{12}{6,000}$, which reduces to $\frac{1}{500}$, which we can also write as $\frac{2}{1,000}$; and in decimal form this can be written as 0.002**

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[2]It's similar to saying that $30 \div 10$ doesn't look like $6 \div 2$ but they both namje the same rate.  For example at a rate of 6 for \$2 you woulod get 30 for \$10.

### Practice Problem #5

**Write the quotient 0.02 ÷ 0.0004 as a whole number**

**Answer:  50**

### Solution*:*

To convert 0.0004 to a whole number, we have to move the decimal point 4 places to the right.  This means we are multiplying 0.0004 by 10,000.  And if we multiply 0.0004 by 10,000 we also have to multiply 0.02 by 10,000 in order not to change the ratio  In other words, we move the decimal point 4 places in both 0.0004 and 0.02 to obtain the equivalent ratio 200 ÷ 4, which is 50.

As a check we notice that 50 × 0.0004 = 0.02

### *Notes*:

• What the answer means is that as small as 0.02 is it is still 50 times as great as 0.0004.

• We could move the decimal point even more places to the right but it wouldn't affect the answer.  For example, if we move the decimal point 6 places to the right in both numbers the ratio would be 20,000 ÷ 400, or still 50.

• If we wanted to use the fraction format for division we could have rewritten the problem as $\frac{0.02}{0.0004}$; after which we could multiply numerator and denominator by 10,000 to obtain:

$$\frac{0.02 \times 10,000}{0.0004 \times 10,000} = \frac{200}{4} = 50.$$

Up to now our trip through the world of decimal fractions may not have seemed overly exciting.  When all is said and done, it seems that except for learning where to put the decimal point and how many places to move it, we do decimal arithmetic the same way that we do whole number arithmetic.  However, as we shall discuss in the nest part of this lesson, there is more to the division of decimals than what first meets the eye. *Namely, when we try to represent the quotient of two whole numbers as a decimal fraction, more often than not, the resulting decimal <u>never</u> comes to an end.*