Unedited Draft

Arithmetic Revisited

Lesson 5:

Decimal Fractions or Place Value Extended

Part 3: Multiplying Decimals

1. Multiplying Decimals

Multiplying two (or more) decimals is very similar to how we use place value to multiply two or more whole numbers. In fact, what we shall show is that:

- **we obtain the “adjective” part of the answer by multiplying as if there were no decimal points;**

  -- In this context, the adjective part of the product depends only on the adjectives of the numbers being multiplied. In short, the decimal point does not affect the digits we obtain as the product

- **and we use the decimal points to determine the denomination that the adjective modifies.**

  -- That is, the only purpose served by the decimal points is to determine the noun that is modified in the product

Let's illustrate what we mean by looking at particular illustration.

Illustrative Example #1:

Write the product 0.03×0.002 as a decimal fraction.

Method 1:

Because we already know how to multiply common fractions, a good strategy might be to rewrite each decimal as an equivalent common fraction. Remembering that the number of digits to the right of the decimal point tells us the number of 0's in the denominator, we see that:

-- 0.03 means $\frac{3}{100}$ and 0.002 means $\frac{2}{1,000}$. Hence

$$0.03 \times 0.002 = \frac{3}{100} \times \frac{2}{1,000} = \frac{6}{100,000}$$

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We now have the correct answer as a common fraction but we want to express the answer as a decimal fraction. We see that the denominator on the right hand side of the above computation has five zeros, which tells us that there must be five digits to the right of the decimal point in our answer. That is,

$$0.03 \times 0.002 = 0.00006$$

We read 0.00006 as 6 hundred-thousandths. Namely:

1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$	$\frac{1}{1,000,000}$
	.	(0)	(0)	(0)	(0)	6	

***Aside*:**

Several times in this course we have seen that when we multiply two numbers, we multiply the two adjectives to get the adjective of the product and we multiply the two nouns to get the noun of the product. We used this result as a way of showing why we count the number of zeroes when we multiply, say 200 by 3,000. More specifically, if we think of 200 as being 2 hundred and 3,000 as being 3 thousand, we obtain:

$$\begin{array}{cccccc} 300 & \times & 2{,}000 & = & \\ 3 \text{ hundred} & \times & 2 \text{ thousand} & = & \\ (3 \times 2) & \times & (\text{hundred} \times \text{thousand}) & = & \\ 6 & \times & \text{hundred thousand} & = & 600{,}000 \end{array}$$

The same reasoning can be applied almost verbatim to the given problem. That is:

$$\begin{array}{cccccc} 0.03 & \times & 0.002 & = & \\ 3 \text{ hundredths} & \times & 2 \text{ thousandths} & = & \\ (3 \times 2) & \times & (\text{hundredths} \times \text{thousandths}) & = & \\ 6 & \times & \text{hundred thousandths} & = & 0.00006 \end{array}$$ [1]

[1]Notice the following similarities: 3 ***hundred*** × 2 ***thousand*** = 6 ***hundred thousand***
3 ***hundred<u>ths</u>*** × 2 ***thousand<u>ths</u>*** = 6 ***hundred thousan<u>dths</u>***

Method 2:

This method can be viewed as an outgrowth of Method 1. More specifically:

- **Notice that the numerator of $\frac{6}{100{,}000}$ was obtained by multiplying as if there were no decimal points in the given problem (that is, $3 \times 2 = 6$).**

- **The denominator consists of a 1 followed by as many 0's as there are digits to the right of the decimal points.**

0.03. 0.002.
12 345

-- The reason for this is that when we translate a decimal fraction to a common fraction we get a zero in the denominator for each digit that was to the right of the decimal points in the decimal fractions.

Note:

0's can be omitted when they are unimportant but not when they are important. When in doubt don't omit the 0's. For example, if the problem had been 0.05×0.002, we would have multiplied 5 by 2 to get 10. There are now *two* digits to the left of the decimal point (0 *is* a digit). Hence when we move the decimal point 5 places to the left, we only have to annex *three* 0's; thus obtaining 0.00010. This is a correct answer but since the final 0 doesn't effect the place "1" is in, we may "drop" it and write the answer as 0.0001.

If we generalize the procedure we used in Illustrative Example 1, we see that to multiply any two decimals:

***Step 1*:**

Multiply the decimals as if there were no decimal points, but remembering that a decimal point is assumed to be immediately after the digit that's furthest to the right (the 1's place).

***Step 2*:**

Then count the total number of digits (in both factors) that are to the right of the decimal points.

***Step 3*:**

Whatever number you obtained in Step 2, move the decimal point in the product the same number of places to the left.

Practice Problem #1

A student, using a calculator, multiplies 3.14 by 2.7 and obtains 84.78. By using estimation, how might the student know that an error was made?

Solution:

One such way is to observe that $3.14 < 4$ and $2.7 < 3$. Hence

$$3.14 \times 2.7 < 4 \times 3$$

While there are many numbers that are less than 12, 84.78 isn't one of them!

Notes:

• In terms of our area model, the rectangle whose dimensions are 3.14 feet by 2.7 feet fits inside the rectangle whose dimensions are 4 feet by 3 feet. In turn, the rectangle whose dimensions are 3 feet by 2 feet fits inside both rectangles. More symbolically:

$$\begin{array}{rccccc} & 3 & < & 3.14 & < & 4 \\ \times & 2 & < & 2.7 & < & 3 \\ \hline & 6 & < & 3.14 \times 2.7 & < & 12 \end{array}$$

• In other words, before we begin to compute the exact value of 3.14×2.7, it is helpful to know that the correct answer must be between 6 and 12.

Practice Problem #2

Express the product 0.043×0.02 as a decimal fraction

Answer: 0.00086

Solution:

The cut-and-dried method is to pretend the decimal points are not there and multiply the resulting whole numbers. In this case we obtain $43 \times 2 = 86$.

We then count the total number of digits to the right of the decimal points in both factors. There are three in 0.043 and two in 0.02; for a total of 5. Hence we move the decimal point in 86. five places to the left to obtain the answer:

Notes:

- **There are several ways to obtain the answer more logically.**

-- One such way is to convert the decimal fractions into common fractions. Thus: 0.043 $= \frac{43}{1,000}$ and 0.02 $= \frac{2}{100}$. Hence:

$$0.043 \times 0.02 = \frac{43}{1,000} \times \frac{2}{100}$$

$$= \frac{43 \times 2}{1,000 \times 100}$$

$$= \frac{86}{100,000}$$

= 86 hundred-thousandths

= 0.00086

- **Another way is to use the adjective/noun theme. For example, if the problem had been**

43 thousand $\times$ 2 hundred,

the answer would have been

86 hundred thousand

In an analogous way if the problem is

43 thousandths $\times$ 2 hundredths,

the product is

86 hundred thousandths

2. Notes on Multiplying and Dividing Decimals by Powers of 10

There is a connection between the way we use the decimal point for decimal fractions and 0's for whole number powers of ten. For example, just as we annex two's 0's to take the place of the word "hundreds" we move the decimal point two places to the *left* to take the place of the words "hundredths".

-- Notice that annexing two 0's to whole number when we multiply it by 100 is the same as moving the decimal point two places to the right. Namely, if there is no decimal point the 1's place is indicated by the digit that is furthest to the right. Hence annexing two 0's is equivalent to moving the decimal point two places to the right. In other words, when we annex two 0's to 3, it converts 3. into 3$\underset{1}{0}\underset{2}{0}$.

-- More generally, to multiply by 10 we move the decimal point one place to the right; and because 100 $=$ 10 $\times$ 10, to multiply by 100, we move the decimal point two places to the right, etc.[2]

-- In a similar way, to divide by 10 we move the decimal point one place to the left; to divide by 100 we move the decimal point to places to the left, etc.

-- A rather easy way to remember this is in terms of money. For example, suppose 100 people share equally the cost of a $283 gift. Then:

$283 $\div$ 100 $=$ $2.83, which is the price each one pays; and

$2.83 $\times$ 100 $=$ $283.

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[2] When we annex two 0's to a whole number we are, in effect, moving the decimal point two places to the right. However, we prefer to emphasize moving the decimal point rather than annexing 0's in the hope of avoiding some possible confusion. For example, to multiply 0.38 by 100 we move the decimal point two places to the right to obtain 38. If we had merely annexed two 0's to 0.38, we would have obtained 0.3800, which is an equivalent way of saying 0.38. In other words, 0.38 $=$ 0.3800.

<u>Key Point</u>:

If n is any whole number, to *multiply* any decimal by 10^n, we move the decimal point n places to the *right*. To *divide* a decimal by 10^n, we move the decimal point n places to the *left*.

Thus another way of saying "Move the decimal point n places to the right" is "Multiply the decimal by 10^n" and another way of saying "Divide a decimal by 10^n" is "Move the decimal point n places to the left"

In the next part of this lesson we will begin a discussion of how we divide one decimal by another.