CALCULUS IN EVERYDAY LIFE

PART 1

INTRODUCTORY OVERVIEW

(A Non-Mathematical Approach)

1. A Special Note to Teachers

It is not uncommon for elementary school teachers in a mathematics workshop to ask such questions as "Why are we discussing how to draw the graph of x + y = 10? We don't teach this in elementary school". Yet if they are discussing a work of Shakespeare in a literature workshop, they seldom, if ever, ask "Why are we discussing the works of Shakespeare? We don't teach this in elementary school".

The point is that whether it is mathematics or any other topic, things we know at the advanced level often give us insight to what we should teach at the more elementary levels. So, for example, if first grade teachers understand what it means to draw the graph of x + y = 10, they can ask such classroom questions as "Can you name two numbers that add up to 10?" And If a student answers "4 and 6", the teacher could then ask "Can someone name two other numbers that add up to 10?". In this way students, even at the first grade level, realize that there are several ways that two whole numbers might add up to 10 even though they might have no idea of what is meant by $x + y = 10$.

At the first grade level, students might justifiably view the word "number" as meaning "whole number" but as they progress from grade to grade, the same question might lead to students naming such additional number pairs as "12 and $^{-}2$", "7.6 and 2.4", etc. In that way by the time these students get to the place in the curriculum where they are asked to draw the graph of $x + y = 10$, they are already familiar with many of the steps that the answer requires.

In a similar way, the study of calculus involves, among other things, the concept of instantaneous rates of change. Except for the concept of "instantaneous", the concept of rate of change is studied extensively in the K-8 mathematics curriculum (in the form of fractions in elementary school and in the form of linear functions in middle school). Thus by properly internalizing the K-8 curriculum, students are well on their way to the study of calculus.

2. Preface to All Readers:

Reinforcement of the importance of the role that arithmetic and algebra play in preparing students to take calculus is explained quite elegantly by Ken Gross. Ken, along with his brother Herb, developed the Vermont Mathematics Initiative (VMI) and uses "Calculus for K-8" as the capstone course for his program. The preamble to this course, as written by Ken, reads:

> **"Over the years I have taught calculus at many universities (too many universities), and have observed the following sad outcome many times (too many times).**
>
> - **A hard working motivated student wants to be an engineer.**
> - **Calculus becomes the obstruction to the student's aspirations.**
> - **The student does poorly in calculus, and ultimately switches major to some non-science discipline.**
>
> **In actuality, the student did not fail calculus, but algebra failed the student.**
>
> **More specifically, when you look beneath the surface, algebra was not the problem. The student's understanding of and skill with arithmetic were inadequate.**
>
> **SUMMARY:**
>
> **The student failed calculus because, years earlier, he/she had failed arithmetic, possibly without knowing it.**
>
> **This, in part, is the reason "calculus K-8" is chosen as the capstone mathematics content course for VMI. The K-8 teacher has more to do with the college student's success in calculus than does the calculus instructor!"**

In the spirit of Ken's preamble we might have elected to call this course "Calculus for Elementary and Middle School Teachers". However we have elected to call our capstone course "Calculus in Everyday Life" not only to show other interested viewers how the mastery of basic arithmetic and elementary algebra serves as the basis for unraveling the mysteries of calculus, but also to highlight how the underlying principles in calculus are present in our daily life.

To be sure, by the end of this course you should be able to answer such questions as:

(1) A rectangle is to be formed by using a piece of wire that is 36 inches long. What will its dimensions be if it encloses the greatest area?

(2) A ball is projected vertically upward so that it reaches a height of h feet after t seconds where $h = 96t - 16t^2$. At what time will the ball be at its greatest height?

(3) An object moves along the x-axis according to the rule $x = t^3$, where x is in feet and t is in seconds. How fast is the object moving when the time is exactly 4 seconds?

(4) An object, moving along the x-axis, has a speed of v feet per second at the end of t seconds. How far will the object have traveled during the time interval between 4 seconds and 6 seconds?

3. The Concept of an Instant

When I taught calculus, a tongue-in-cheek assignment I gave students after the first class meeting was to watch the grass grow and then write an essay on why the lawn had to be mowed every week. In other words at any given instant of time the grass did not appear to be growing and it is when we try to come to grips with what we mean by an *instant* that the study of calculus begins.

Perhaps our above discussion might be easier to visualize in terms of watching a motion picture film.

-- Imagine, for example, an unraveled reel of movie film. The first and second frames, especially to the naked eye, look identical. So do the second and the third frames. We can continue in this way, looking at frame after frame, without noticing any difference between any two *consecutive* frames. However, by the time we get to the fifty thousandth frame, we see that there is a huge difference between that frame and the first frame.

-- In other words, while, say, the 347^{th} frame and the 348^{th} frame are virtually indistinguishable, the subtle difference plays a role in the huge difference that takes place between the 1st frame and the $50{,}000^{th}$ frame. In more poetic terms, each frame, in its millisecond of existence, exerts an influence on every subsequent frame! In essence, a very large number of small changes can produce a big change overall.

In a similar way we may compare life to the reel of movie film with each frame representing an instant. Except for a relatively few days in which extraordinary things may happen[1], we notice very little change on a day-to-day basis[2].

Here is another example: Often when there is a huge snow storm almost invariably there is an older person who says “This is nothing compared to the storms when I was a kid. I remember that the snow drifts were over my head!" This was indeed true but the person forgets that he might have been barely taller than 3 feet! What happened is that his height changed so slowly that his perception is that the snow drifts became less high.

4. The Calculus of Change:

For the most part automobile design didn't change much between, say, 1975 and 1976; nor did it change much between 1976 and 1977; and so on through the years until we got to the year 2000. Yet when we look at two cars, one from the year 1975 and the other from the year 2000, the two cars seem to be very much different. It is difficult, if not impossible, to determine the year in which the major change took place.

In a similar way do you ever recall a day when you said something similar to "Wow! I got much older today!" or "Wow! I got much heavier today" or in school saying "Wow, I got much smarter today!" Yet in the course of many days we do begin to notice such changes.

These subtle changes play a huge role when we try to make changes in society. For example suppose it is the year 1975 and you yearn to be a social reformer. You see something that you believe is wrong and you vow that when you become powerful enough to make a difference, you will make some important changes. It is now 25 years later and you are now in such a position and you turn your attention to the problem that first began to bother you in 1975. On a day-to-day basis life didn't change a lot in the intervening years, thus the chances are good that you remember the situation as it was 25 years ago. You have forgotten that many significant changes have occurred in society during the 25 years that have elapsed since you noticed the problem. So without realizing you solve the problem as if it was still the year 1975. In still other words, you solved the correct problem but for the wrong generation.

---

[1]In science we refer to such days as perturbation points. A perturbation point is when a small change in one variable produces a very large straw in another variable. In essence, a perturbation point might be the straw that broke the camels's back, If there were no such point we could load an infinite amount of straw on a camel's back, just by doing it one straw at a time.

[2]In terms of our roll of movie filma analogy, on those days when a huge change takes place it would be equivalent where film had been spliced and there was a huge change in going form one frame to the enext frame.

For example, in many communities where crime has dramatically increased over a period of, say 25 years, there is a proposal to bring back the "good old days" when a police officer had a beat and knew the people on his beat and the people knew him quite well also. There is a tendency to forget how much cities and towns have grown since the days of the "beat cop" and it is quite likely that a town could use up its allotment for police patrols just to ensure there were enough police officers patrolling the many town malls, etc. What really has to happen is for people to get together and ask such questions as "What do we have to do today to solve the problems that were solved by having police officers patrol a beat 25 years ago?

Similar problems arise with respect, say, to education. We often hear remarks that begin with "When I was in school...." and we forget that the world has changed since then. In fact in my own situation the traditional family of my generation was the nontraditional family of the next generation. When I went to school my mother was at home, making sure I did my homework before I went out to play. As little as one generation later, most mothers were part of a two income family and weren't at home when their children came home from school. In too many situations there was no one at home to make sure that the children did their homework before they went out and played. Yet we tend to blame teachers for not doing as a good job as their predecessors did rather than to blame a society in which both parents were working and often too tired to work with their children when they get home from work.

The list goes on and on but this is not a sociology course! Our point is simply that because time transpires one instant at a time we often fail to notice changes until they become virtually overwhelming.

5. The "Quick Fix" Fallacy (A Highly Subjective Overview):

If we gained weight in, say, ten pound increments the chances are that there would be far fewer obese people. The fact is that we usually gain weight a few ounces at a time and that by the time we first notice it, chances are that it has been going on for a very long time.

The problem is that because we just noticed it, we think that it must have happened very recently and therefore it shouldn't take long to remedy the situation. So we diet for an entire weekend (not just one day!) and we are dismayed on Monday morning when we notice that there has been no significant weight loss.

In terms of our unraveled-roll-of-movie-film analogy, there has been a weight loss but it is confined to just a few frames in the film of our daily life. The problem is that we too often become impatient and dismiss the notion of remaining on a diet.

Our light hearted description above of what it means to look for a "quick fix" extends to how we too often handle more serious problems that occur

in the socioeconomic-political arenas. Namely by the time the particular problem becomes noticeable it has most likely been festering for a long time; but because we just noticed it, we expect that it shouldn't take long to solve. And when the problem isn't solved in what we feel is a reasonable amount of time, we too often feel that it is a waste of time, money and effort to continue our attempted solution.

There are many real-life examples we could dwell on with respect to the "quick fix" myths but at the moment it might be wise to restrict our discussion to the logic that went into the design of our presentations on our website.

One of our favorite adages is "Never make the same mistake once". In other words, if nothing is done incorrectly there is nothing that has to be remediated. Thus our approach to mathematics education has been to start at the very beginning, some referred to as "the dawn of consciousness". In a step-by-step approach, using our adjective/noun theme, we began with hieroglyphics and proceeded in a logical way to the development of the real number system and beyond

In our development we showed how arithmetic and algebra were in essence two sides of the same coin and how the concept of paraphrasing allowed us to get from arithmetic to algebra by way of a seamless transition.

We are now about to embark on a similar quest in which we shall show how our study of arithmetic and algebra, coupled with a mathematically precise way to capture the meaning of an "instant", is all we need to begin a trip into the world of calculus.